Importance Function Derivation for RESTART Simulations of Petri Nets

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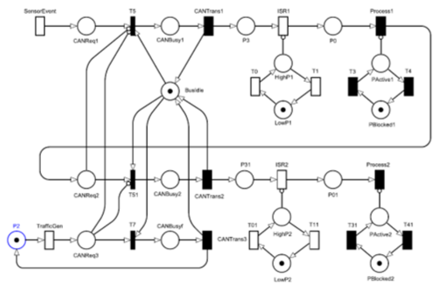
Outline

- Stochastic Petri Nets
- RESTART Simulation
- A-Priori Estimation of Rare Event Frequencies
- A Heuristic Importance Function for SPNs
- An Example
- Conclusion

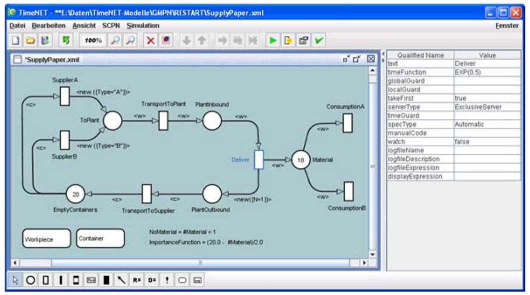
Introduction

- **Accessible Rare-Event Simulation**

  - Underlying model
    - Stochastic Petri nets



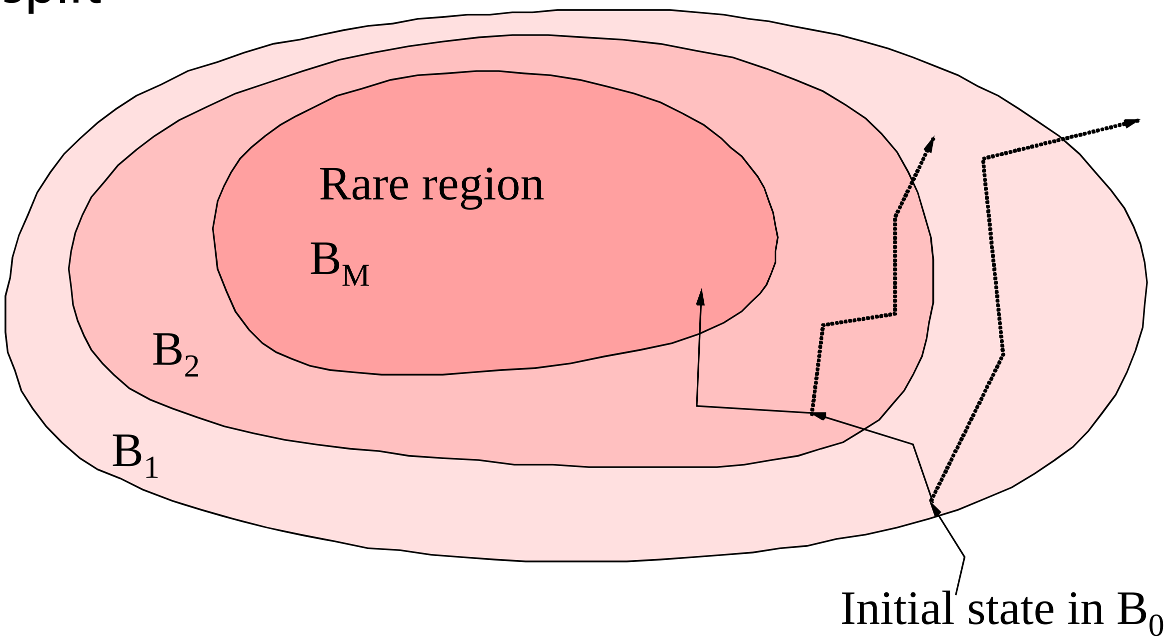
  - Software Tool
    - TimeNET



  - Less expert knowledge
    - Semi-automatic parameter setting

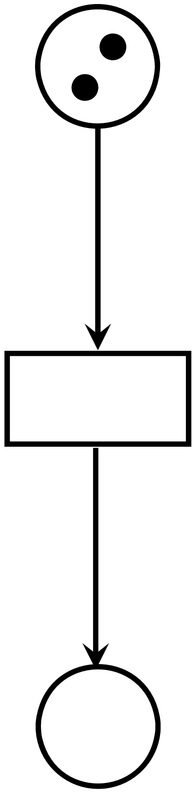
RESTART

- **Splitting Simulation**
  - Model and simulation loop as (semi-) black box
  - Controlling RESTART algorithm only needs to …
    - Store and recall state
    - Decide when to split and how often
    - Technical: **Importance function** and **Levels**
    - Crucial for speedup!



Stochastic Petri Nets

- **Model Elements**
  - **Places** P – state variables
    - Marking – state, initial: $\mathbf{m}_0$
  - **Transitions** T – events
    - Input arcs $\mathbf{Pre} : P \times T \to \mathbb{N}$
    - Arc multiplicities
    - Enabling and firing
  - **Performance measures**
    - Rate and impulse rewards
    - Concentrate on rare transition t_{rare} firing frequency γ



Stochastic Petri Nets

- **Algebraic Description**
  - **Marking** vector $\mathbf{m} \in \mathbb{N}^{|P|}$ and direct reachability $\mathbf{m} \xrightarrow{t} \mathbf{m}'$
  - **Incidence matrix** $\mathbf{C}$ with dimensions $|P| \times |T|$
    $\mathbf{C}(p,t)$ denotes token change in p when t is fired
  - **Next marking** $\mathbf{m}' = \mathbf{m} + \mathbf{C} \cdot (0, \ldots 0, 1_t, 0, \ldots 0)$
  - **State equation** $\mathbf{m} = \mathbf{m}_0 + \mathbf{C}\,\sigma$
    for a firing count vector $\sigma \in \mathbb{N}^{|T|}$
  - **Invariants** (P- and T-semiflows)

$$\mathbf{y}\,\mathbf{C} = 0 \quad \longrightarrow \quad \mathbf{y}\,\mathbf{m} = \mathbf{y}\,\mathbf{m}_0 + \mathbf{y}\,\mathbf{C}\,\sigma = \mathbf{y}\,\mathbf{m}_0 = \text{const.}$$

$$\mathbf{C}\,\mathbf{x} = 0 \quad \longrightarrow \quad \mathbf{m}' = \mathbf{m} + \mathbf{C}\,\mathbf{x} = \mathbf{m}$$

A-Priori Estimation

- **Estimation of Performance Measures**
  - [Campos et. al.]
  - Structural analysis of model
    ⇨ constraints for upper and lower bounds of results
  - Based on incidence matrix and other model information
  - Formulation as a linear programming problem

  - Notation: firing frequency = throughput
    = inverse of average interfiring time

$$\gamma = \chi[t_{rare}] = \frac{1}{\Gamma[t_{rare}]}$$

A-Priori Estimation

- **Visit Ratios and Service Demands**
  - Derive **relative visit ratios** $\mathbf{v}^{(1)} : T \to \mathbb{R}^+$ of transitions using incidence matrix, transition invariants, and firing probabilities of conflicting transitions, relative to t_{rare}
  - Define **average service demands** $\overline{\mathbf{D}}^{(1)} : T \to \mathbb{R}^+$ relative to a transition by multiplying visit ratios by a transition's mean service time $\overline{s}[t_i]$ (its delay)

$$\overline{\mathbf{D}}^{(1)}[t_i] = \mathbf{v}^{(1)}[t_i]\,\overline{s}[t_i]$$

A-Priori Estimation

- **LPP Formulation for Bounds**
  - Apply **Little's Law** to each (timed) transition and its input places for ($\overline{\mathbf{m}}$ denotes the mean marking vector)

$$\Gamma[t_{rare}]\,\overline{\mathbf{m}} \geq \mathbf{Pre} \cdot \overline{\mathbf{D}}^{(1)}$$

  - and thus

$$\Gamma[t_1] \geq \max_{\mathbf{y} \in \{\text{P-semiflows}\}} \frac{\mathbf{y} \cdot \mathbf{Pre} \cdot \overline{\mathbf{D}}^{(1)}}{\mathbf{y} \cdot \mathbf{m}_0}$$

  - Leading to a **bound** for the **mean interfiring time**

$$\begin{array}{rlrcl} \Gamma[t_{rare}] = & \text{maximum} & \mathbf{y} \cdot \mathbf{Pre} \cdot \overline{\mathbf{D}}^{(1)} & & \\ & \text{subject to} & \mathbf{y} \cdot \mathbf{C} & = & 0 \\ & & \mathbf{y} \cdot \mathbf{m}_0 & = & 1 \\ & & \mathbf{y} & \geq & 0 \end{array}$$

A-Priori Estimation

- **Throughput Estimation**
  - **Throughput upper bound**

$$\chi_+[t_{rare}] = \frac{1}{\Gamma[t_{rare}]}$$

  - **Throughput lower bounds**: worst-case interfiring time after all other transitions have fired using visit ratios

$$\begin{aligned}\chi_-[t_1] &= \frac{1}{\sum_{t \in T} \overline{\mathbf{D}}^{(1)}[t]} \\ \chi_-[t_j] &= \chi_-[t_1]\,\mathbf{v}^{(1)}[t_j]\end{aligned}$$

  - **Heuristic estimate**: weighted average of bounds

$$\gamma = \chi[t_{rare}] \approx \alpha\chi_-[t_{rare}] + (1-\alpha)\chi_+[t_{rare}]$$

Heuristic Importance Function

▪ State Distances in Petri Nets

- Following similar ideas, the minimal number of events (transition firings) $h(\mathbf{m}, t_{rare})$ to reach a marking $\mathbf{m}'$ enabling transition t_{rare} is

$$
\begin{array}{lrcl}
\text{minimize} & h(\mathbf{m}, t_{rare}) & = & \mathbf{x}\,\mathbf{1} \\
\text{subject to} & \mathbf{m}' & = & \mathbf{m}_0 + \mathbf{C}\,\mathbf{x} \\
 & \mathbf{x} & \geq & 0 \\
 & \mathbf{m}' & \geq & \mathbf{Pre}(\cdot, t_{rare})
\end{array}
$$

- **Admissible** and **monotone** ⇨ useful for search problems
- Can be generalized to general marking conditions

Heuristic Importance Function

- **Importance Function Definition**
  - Let $h_{\max} = h(\mathbf{m}_0, t_{rare})$, the maximum* state distance
  - **Importance function** for marking $\mathbf{m}$:

$$f_I(\mathbf{m}) = h_{\max} - h(\mathbf{m}, t_{rare})$$

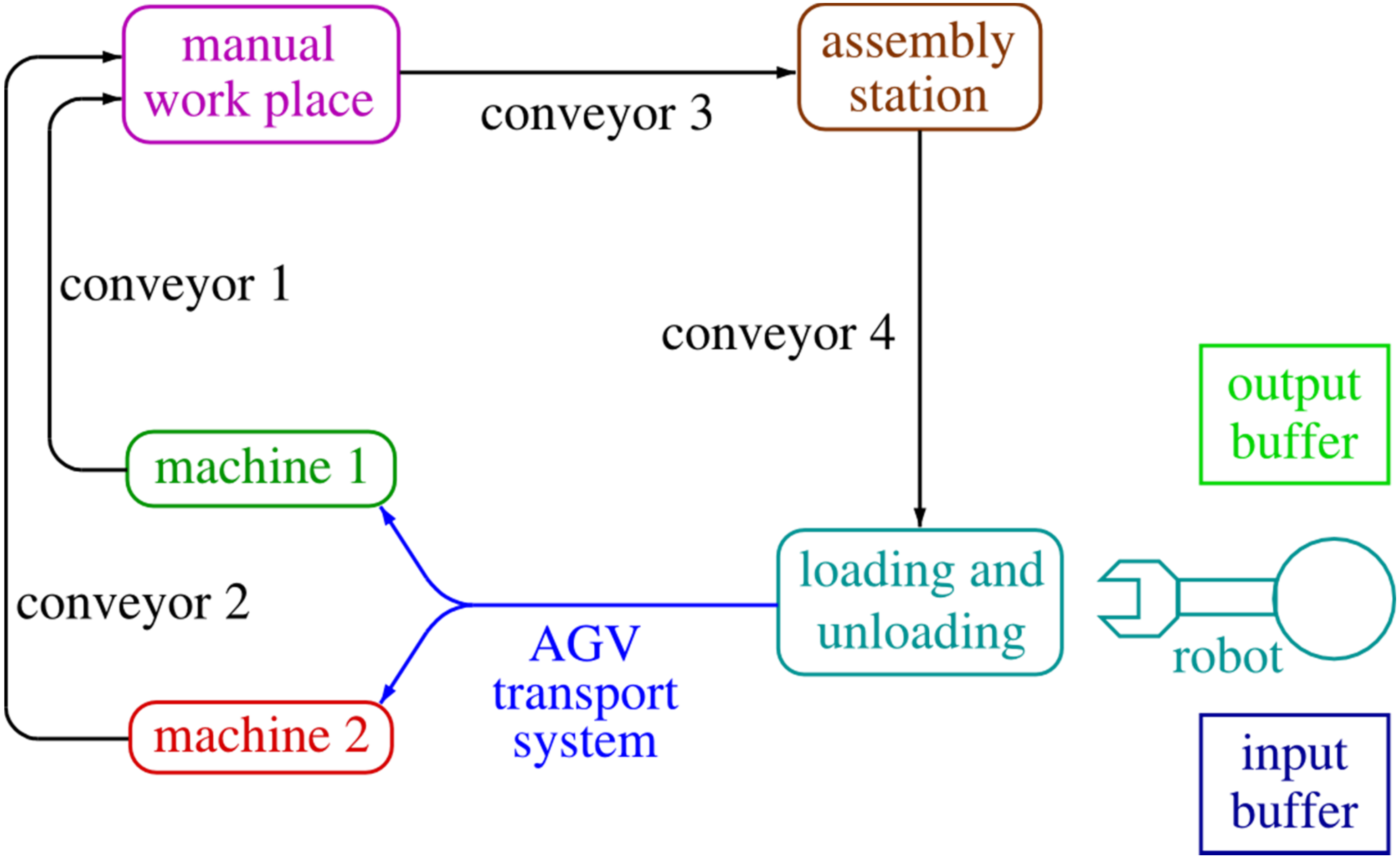
  - #Levels, optimal: $L_{\text{opt}} = \frac{\ln \gamma}{\ln p_k}$ with $p_k \in \left[\frac{1}{2}, e^{-2}\right]$

    maximum: $L = \min(L_{\text{opt}}, h_{\max})$

  - **Threshold setting**: $Thr_i = \left\lfloor i \, \frac{h_{\max}}{L} \right\rfloor$

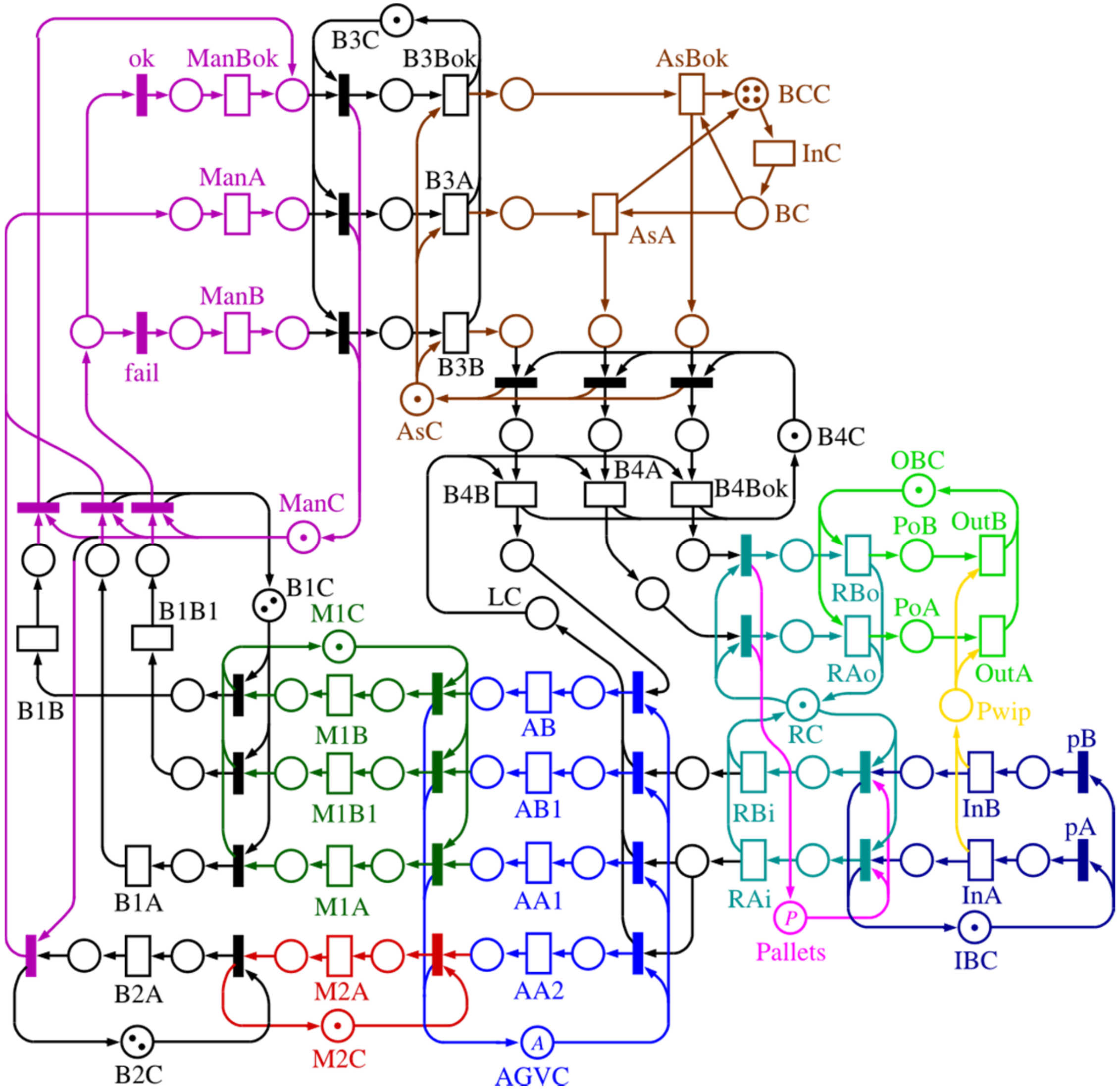
Example

- **Flexible Manufacturing System**



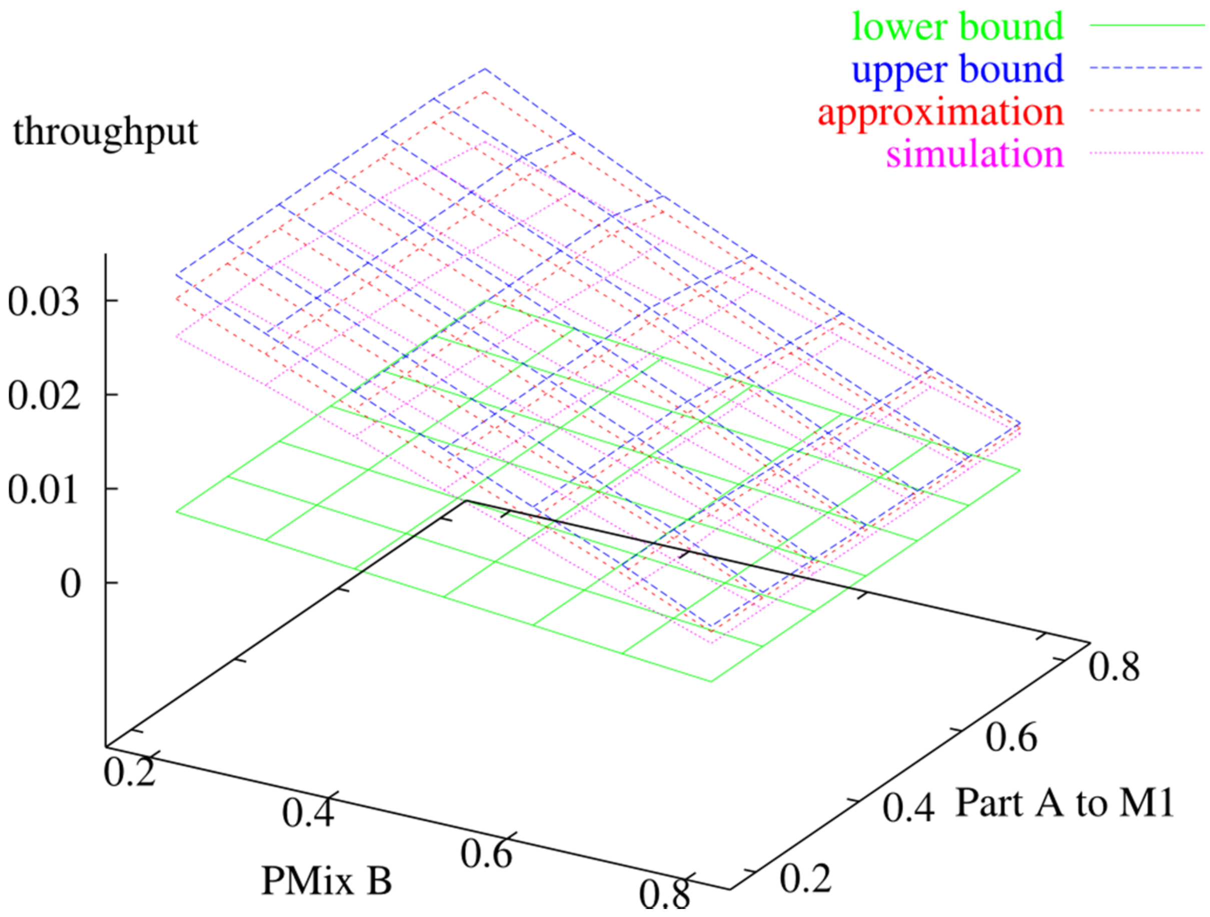
Example

- **Stochastic Petri Net (GSPN)**
  - Different parts in various states



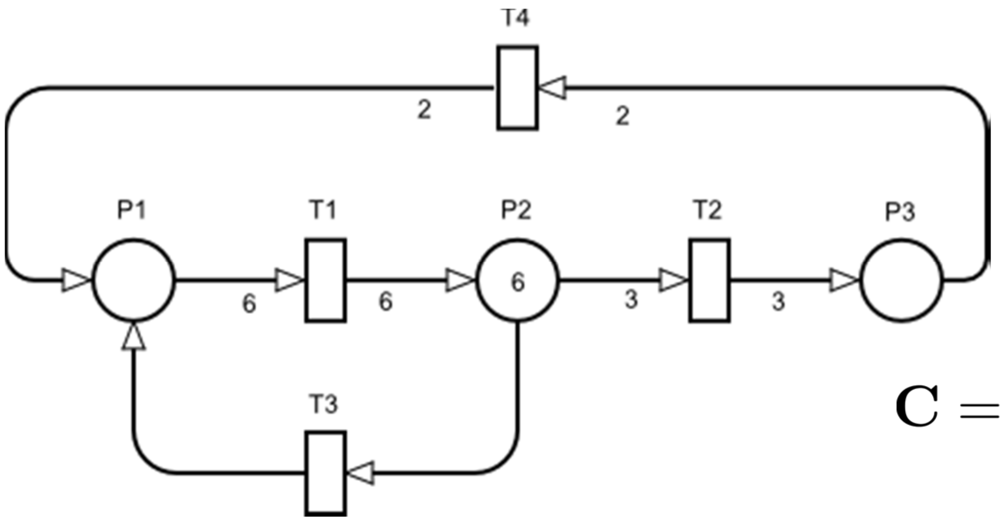
Example

- **Throughput Estimation Quality**



Example

- **State Distance Heuristic**
  - Simple model (manual solution)



$$\mathbf{C} = \begin{pmatrix} -6 & 0 & 1 & 2 \\ 6 & -3 & -1 & 0 \\ 0 & 3 & 0 & -2 \end{pmatrix}$$

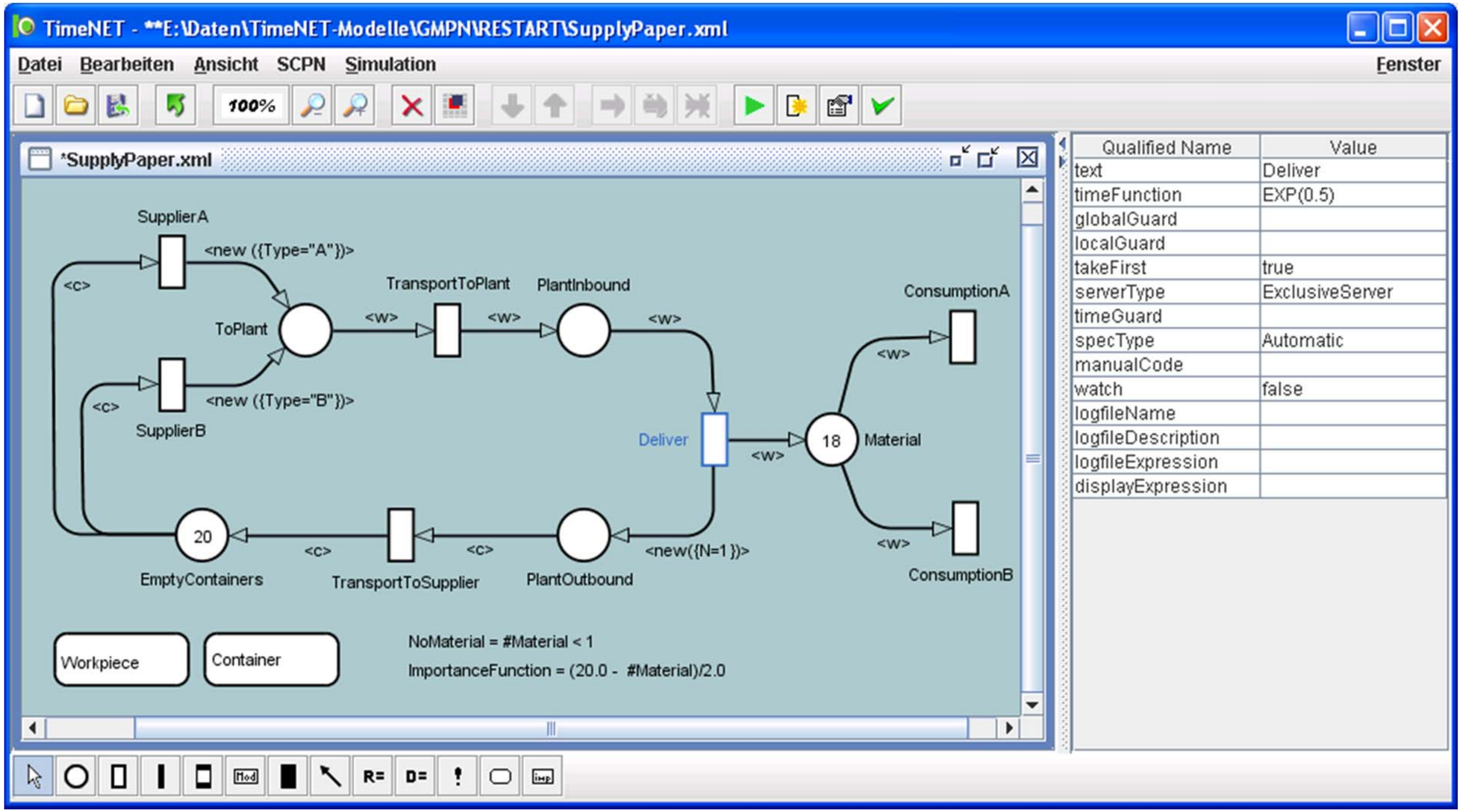
  - How many transition firings until T1 is enabled?
    5: 2x T2, 3x T4
    (if arc multiplicities of T2 or T4 are decreased, then 6x T3)

Conclusion

- **Improvement of RESTART usability for SPNs**
  - A-priori estimation of rare event frequency
    ⇨ **Optimal number of thresholds**
  - Heuristic state distance measure
    ⇨ **Importance function**

- **Open Issues**
  - Estimate other types of performance measures
  - Guidance quality of state distance heuristic
  - Tool implementation and complex examples
  - Computational effort?

Tool TimeNET



Stochastic Petri net tool – available at **www.tu-ilmenau.de/sse/TimeNET**